AP Calculus

Advance Mathematics beyond Common Core Curriculum Standards

“Calculus need not be made easy, it is easy.”

Jaime Escalante
COURSE OVERVIEW

Introduction

AP Calculus is the study of limits, derivatives and integrals through the Fundamental Theorem of Calculus and some applications of the definite integral. This study corresponds to slightly more than the first semester course taught at many colleges and universities. The AP Calculus course is designed as a full year, challenging course on limits, derivatives and integrals which will focus on real world methodology and application and lead the successful student to a working knowledge of single variable Calculus. The highly successful student will have the knowledge to demonstrate their ability on the AP Calculus exam. Students who elect to may receive college credit through the CAP program at BCC. This course requires that students will have successfully completed Precalculus (B or better) and are capable of self motivation and direction. All students will have and use a graphing calculator on a daily basis. While most students have their own, the school issues graphing calculators for those students who do not. Given the successful completion of this course, students will be prepared to undertake a college level Calculus course beginning with the 2nd level.

By successfully completing this course, the student will be able to:
• Work with functions represented in a variety of ways and understand the connections among these representations.
• Understand the meaning of the derivative in terms of a rate of change and local linear approximation, and use derivatives to solve a variety of problems.
• Understand the relationship between the derivative and the definite integral.
• Communicate mathematics both orally and in well-written sentences to explain solutions to problems.
• Model a written description of a physical situation with a function, a differential equation, or an integral.
• Use technology to help solve problems, experiment, interpret results, and verify conclusions.
• Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
• Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

A Balanced Approach

Current mathematical education emphasizes a “Rule of Four.” There are a variety of ways to approach and solve problems. The four branches of the problem-solving tree of mathematics are:

• Numerical analysis (where data points are known, but not an equation)
• Graphical analysis (where a graph is known, but again, not an equation)
• Analytic/algebraic analysis (traditional equation and variable manipulation)
• Verbal/written methods of representing problems (classic story problems as well as written justification of one’s thinking in solving a problem—such as on our state assessment)

Technology Requirement

A Texas Instruments 84 Plus graphing calculator in class regularly. The calculator in a variety of ways including:
• Conduct explorations.
• Graph functions within arbitrary windows.
• Solve equations numerically.
• Analyze and interpret results.
• Justify and explain results of graphs and equations.

Please note, students selecting this Calculus course should have already met the college and career ready standards plus standards marked with a +. AP Calculus is not covered by the Common Core Standards as it exceeds the requirements expected in the high school curriculum. This course is certified by Burlington County Community College as meeting the requirements for CAP credit and is a dual enrollment course. This course is certified by the AP College Board as meeting the requirements for the AP Exam in Calculus AB.

Several weeks in May will be devoted to reviewing and preparing students to take the AP Calculus exam. Following the AP exams, students will be engaged in a long term project involving parametric and polar analysis, a topic not normally covered in 1st year Calculus.

The Standards for Mathematical Practice remain an integral part of student academic development as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.
Primary Resources:

Textbook
Title: Calculus (Single Variable, Early Transcendentals)
Authors: Finney, Demana, Waits, Kennedy
Publisher: Pearson Education, Inc.
Copyright: 2004

Title: Calculus for AP
Author: Rogawski, Cannon
Publisher: BFW
Copyright: 2012

Supplementary Resources:

In order to develop mastery of stated objectives, the following may be used:
- Extra practice problems
- Reinforcement exercises
- Transparencies
- On-line applets for motion and development problems
- Maintenance of a notebook and organizational material
- Computer assisted instruction
- On-line quizzes and exercises
- Group explorations and chapter projects
- Self assessments
- Graphing calculator explorations
- A large collection of AP Calculus Exam Preparation manuals and flash cards available for home use

Internet Resources:

The website associated with our text (provides dynamic resources including TI graphing calculator downloads, online quizzing, study tips, Explorations and end-of-chapter projects, and InterAct Math tutorials. In addition, students are encouraged to use www.khanacademy.com, a free math video tutoring site. During class time, there is a daily PowerPoint, demonstration videos (www.education.ti.com) and use of Discovery Learning videos for real world applications. All class resources as well as class work, homework and guided notes are available via the electronic gradebook and the class web site (www.burlington-nj.net link Winegar). There are also several college on line learning resources that my students have permission to use to view classroom lectures and/or work sessions for additional insight and learning. These include MIT on line which is an excellent, high level calculus course for entering freshman (ocw.mit.edu).
Mathematics Standards for High School Overview

1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to de-contextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$. 

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8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation \((y - 2)/(x - 1) = 3\). Noticing the regularity in the way terms cancel when expanding \((x - 1)(x + 1), (x - 1)(x + x + 1), \) and \((x - 1)(x^3 + x^2 + x + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
COURSE OUTLINE

Unit P: Introductory Unit- 1 week

Course Overview and Expectations
✓ General information about the AP Calculus AB Exam
✓ Student Resources- text, in class and online
✓ Course Outline
✓ Useful Acronyms

Summer Assignment: The summer assignment available on line is due first day of school. It allows students working alone or in small groups to review and revise the knowledge gained in PreCalculus in Calculus based scenarios. There is no additional learning required but there may be rethinking.

Entry Level Assessment: In class during the first week of school, students will individually take the phil4 AP Calculus AB Entrance Exam. This exam will aid the student and teacher in identifying individual strengths and weaknesses that will need to be addressed during the course of the 1st semester.

Unit 1: Limits and Continuity (3–4 weeks)

✓ Rates of Change
  ▪ Average Speed
  ▪ Instantaneous Speed
✓ Limits at a Point
  ▪ 1 sided Limits
  ▪ 2 sided limits
  ▪ Sandwich Theorem presentation **
  **A Graphical Exploration is used to investigate the Sandwich Theorem. Students graph y1 = x^2, y2 = -x^2, y3 = sin (1/x) in radian mode on graphing calculators. The limit as x approaches 0 of each function is explored in an attempt to “see” the limit as x approaches 0 of x^2 * sin (1/x). This helps tie the graphical implications of the Sandwich Theorem to the analytical applications of it.
✓ Limits involving infinity
  ▪ Asymptotic behavior (horizontal and vertical)
End behavior models
- Properties of limits (algebraic analysis)
- Visualizing limits (graphic analysis)

 ✓ Continuity
- Continuity at a point
- Continuous functions
- Discontinuous functions
  - Removable discontinuity (0/0 form)
  - Jump discontinuity (We look at y = int(x).)
  - Infinite discontinuity

 ✓ Rates of Change and Tangent Lines
- Average rate of change
- Tangent line to a curve
- Slope of a curve (algebraically and graphically)
- Normal line to a curve (algebraically and graphically)
- Instantaneous rate of change

Unit Expectations:

By the conclusion of unit one, the learner will demonstrate an understanding of the behavior of functions. Specific demonstrations of ability include:

- Demonstrate an understanding of limits both local and global.
  - Calculate limits, including one-sided, using algebra.
  - Estimate limits from graphs or tables of data.
- Recognize and describe the nature of aberrant behavior caused by asymptotes and unboundedness.
  - Understand asymptotes in terms of graphical behavior.
  - Describe asymptotic behavior in terms of limits involving infinity. Compare relative magnitudes of functions and their rates of change.
- Identify and demonstrate an understanding of continuity of functions.
  - Develop an intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
  - Understand continuity in terms of limits.
  - Develop a geometric understanding of graphs of continuous functions. (Intermediate Value Theorem and Extreme Value Theorem).

Authentic Assessment:

Students will develop an intuitive understanding of the nature of limits and explore the connection between the graphical, numerical and analytical approaches to determining limits. Students will complete this activity by investigating behavior of several functions numerically using and t-chart and calculator tables and then graphically using calculators. This lab is completed at the beginning of the Limits Unit. After students
have covered the material relating to analytical methods of finding limits, students again revisit exploring the connection between evaluation of limits graphically, numerically and analytically by rechecking their results using Calculus.

Evaluation Criteria:

Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2\textsuperscript{nd} day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.

Note: The AP Exam is given 50\% calculator, 50\% no calculator.

Benchmark 1 (week 6):

Cumulative assessment comprised of 20 multiple choice questions and 4 free response questions covering PreCalculus basics, limits and continuity. All questions are taken from released AP Exams and are part of the AP Preparation. (Jon Ragowski).

Unit 2: The Derivative (5-6 weeks)

✓ Derivative of a Function
  ▪ Definition of the derivative (difference quotient)
  ▪ Derivative at a Point
  ▪ Relationships between the graphs of f and f’
  ▪ Graphing a derivative from data

**An experiment is conducted with students tossing a large ball into the air. Students graph the height of the ball versus the time the ball is in the air. The calculator is used to find a quadratic equation to model the motion of the ball over time. Average velocities are calculated over different time intervals and students are asked to approximate instantaneous velocity. The tabular data and the regression equation are both used in these calculations. These velocities are graphed versus time on the same graph as the height versus time graph.

✓ One-sided derivatives
✓ Differentiability
  ▪ Cases where f'(x) might fail to exist
Local linearity

**An exploration is conducted with the calculator in table groups. Students graph \( y_1 = \text{absolute value of } (x) + 1 \) and \( y_2 = \sqrt{x^2 + 0.0001} + 0.99 \). They investigate the graphs near \( x = 0 \) by zooming in repeatedly. The students discuss the local linearity of each graph and whether each function appears to be differentiable at \( x = 0 \).

- Derivatives on the calculator (Numerical derivatives using NDERIV)
- Symmetric difference quotient
- Relationship between differentiability and continuity
- Intermediate Value Theorem for Derivatives

✓ Rules for Differentiation
  - Constant, Power, Sum, Difference, Product, Quotient Rules
  - Higher order derivatives

✓ Applications of the Derivative
  - Position, velocity, acceleration, and jerk
  - Particle motion
  - L’Hôpital’s Rule

*Although this topic is not on the AP Calculus AB Exam, this allows students to see the connections between derivatives and limits. Also, it provides a useful way to calculate limits both at a point and as \( x \) approaches +/- infinity. This adds to the rigor of the course and the preparedness of students for college-level mathematics courses.

✓ Applications to Economics
  - Marginal cost
  - Marginal revenue
  - Marginal profit

✓ Derivatives of trigonometric functions

✓ Chain Rule

✓ Implicit Differentiation
  - Differential method
  - \( y' \) method

✓ Derivatives of inverse trigonometric functions
Unit Expectations:

By the conclusion of unit two, the learner will demonstrate an ability to recognize, construct and solve derivatives in all 12 families of functions. Specific demonstrations of ability include:

- Explore and interpret the concept of the derivative graphically, numerically, analytically and verbally.
  - Interpret derivative as an instantaneous rate of change.
  - Define derivative as the limit of the difference quotient.
  - Identify the relationship between differentiability and continuity.

- Apply the concept of the derivative at a point.
  - Find the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
  - Find the tangent line to a curve at a point and local linear approximation.
  - Find the instantaneous rate of change as the limit of average rate of change.
  - Approximate a rate of change from graphs and tables of values.

- Interpret the derivative as a function.
  - Identify corresponding characteristics of graphs of $f$ and $f'$.
  - Identify relationship between the increasing and decreasing behavior of $f$ and the sign of $f'$.
  - Investigate the Mean Value Theorem and its geometric consequences.
  - Translate between verbal and algebraic descriptions of equations involving derivatives.

- Demonstrate fluency and accuracy in the computation of derivatives.
  - Find the derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
  - Use the basic rules for the derivative of sums, products, and quotients of functions.
  - Use the chain rule and implicit differentiation.

Authentic Assessment:

Students will determine the rule for taking derivatives of composite functions (chain rule). Students will be given composite functions and asked to guess the derivative and then determine the derivative using a TI-84 calculator. Students will complete the activity by explaining in correct sentences how to take the derivative of any composite function and how to confirm graphically.

Evaluation Criteria:
Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2nd day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.

**Benchmark 2 (Take Home – Thanksgiving Break):**

Cumulative assessment comprised of 20 multiple choice questions and 4 free response questions covering limits, continuity and rules of differentiation. All questions are taken from released AP Exams and are part of the AP Preparation. (Jon Ragowski).

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**Unit 3: Applications of the Derivatives (5-6 weeks)**

- Extreme Values
  - Relative Extrema
  - Absolute Extrema
  - Extreme Value Theorem
  - Definition of a critical point

- Implications of the Derivative
  - Rolle’s Theorem
  - Mean Value Theorem
  - Increasing and decreasing functions

- Connecting f’ and f’’ with the graph of f(x)
  - First derivative test for relative max/min
  - Second derivative
    - Concavity
    - Inflection points
  - Second derivative test for relative max/min

** A matching game is played with laminated cards that represent functions in four ways: a graph of the function; a graph of the derivative of the function; a written description of the function; and a written description of the derivative of the function.

- Optimization problems
✓ Linearization models
  ▪ Local linearization
    **An exploration using the graphing calculator is conducted in table groups where students graph \( f(x) = (x^2 + 0.0001)^{0.25} + 0.9 \) around \( x = 0 \). Students algebraically find the equation of the line tangent to \( f(x) \) at \( x = 0 \). Students then repeatedly zoom in on the graph of \( f(x) \) at \( x = 0 \). Students are then asked to approximate \( f(0.1) \) using the tangent line and then calculate \( f(0.1) \) using the calculator. This is repeated for the same function, but different \( x \) values further and further away from \( x = 0 \). Students then individually write about and then discuss with their teammates the use of the tangent line in approximating the value of the function near (and not so near) \( x = 0 \).**
  ▪ Tangent line approximation
  ▪ Differentials

✓ Related Rates

Unit Expectations:

By the conclusion of unit three, the learner will demonstrate an ability to recognize, construct and solve derivatives for a variety of applications. Specific demonstrations of ability include:

- Interpret the second derivative.
- Identify the corresponding characteristics of the graphs of \( f, f', \) and \( f'' \).
  - Identify the relationship between the concavity of \( f \) and the sign of \( f'' \).
  - Identify points of inflection as places where concavity changes.
- Apply the derivative in graphing and modeling contexts.
  - Analyze curves, with attention to monotonicity and concavity.
  - Optimize with both absolute (global) and relative (local) extrema.
- Model rates of change, including related rates problems.
- Use implicit differentiation to find the derivative of an inverse function.
- Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- Interpret differential equations geometrically via slope fields and the relationship between slope fields and solution curves for differential equations.

Evaluation Criteria:

Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2nd day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.
Benchmark 3 (Christmas Break):

Cumulative assessment comprised of 20 multiple choice questions and 4 free response questions covering limits, continuity, rules of differentiation and applications of differentiation. All questions are taken from released AP Exams and are part of the AP Preparation. (Jon Ragowski).

Unit 4: The Definite Integral (3-4 weeks)

✓ Approximating areas
  ▪ Riemann sums
  ▪ Left sums
  ▪ Right sums
  ▪ Midpoint sums
  ▪ Trapezoidal sums
  **Here students are asked to input a program that will calculate trapezoidal sums for trapezoids of equal width. They are given this program. They are encouraged to think about altering it to be able to calculate rectangular sums as well.

✓ Definite integrals
  **Students are asked to graph, by hand, a constant function of their choosing. Then they are asked to calculate a definite integral from $x = -3$ to $x = 5$ using known geometric methods. Students then share their work with their tablemates and are asked to come up with a table observation. Those observations are shared with other tables and a formula is discovered.

✓ Properties of Definite Integrals
  ▪ Power rule
  ▪ Mean value theorem for definite integrals
  **An exploration is conducted to show students the geometry of the mean value theorem for definite integrals and how it is connected to the algebra of the theorem.

✓ The Fundamental Theorem of Calculus
  ▪ Part 1
  ▪ Part 2

Unit Expectations:
By the conclusion of unit four, the learner will demonstrate an ability to recognize, construct and solve definite and indefinite integrals using a variety of methods. Students will thoroughly explore the Fundamental Theorem. Specific demonstrations of ability include:

● Explore and interpret the concept of the definite integral.
  ○ Compute Riemann sums using left, right, and midpoint evaluation points.
Find the definite integral as a limit of Riemann sums over equal subdivisions.

Find the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

\[ \int_{a}^{b} f'(x) \, dx = f(b) - f(a) \]

Identify basic properties of definite integrals.

- Apply standard techniques of anti-differentiation.
  - Find anti-derivatives following directly from derivatives of basic functions.
  - Find anti-derivatives by substitution of variables. (including change of limits for definite integrals).

- Apply and interpret the Fundamental Theorem of Calculus.
  - Use the Fundamental Theorem to evaluate definite integrals.
  - Use the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined.

**Authentic Assessment:**

Students will gain an understanding of the geometry involved in the methods of numerical integration. Students will develop the formula for the trapezoid rule. Students will be given problems with graphs with inscribed and circumscribed rectangles drawn. Students will then calculate Riemann sums, make a table or results chart and write a paragraph about the conclusions they have drawn. Students will then rework the given problems by using trapezoids instead of rectangles. Students will show algebraically or geometrically that the Trapezoid Rule is the average of the left hand and right hand Riemann sums.

**Evaluation Criteria:**

Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2nd day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.

**Benchmark 5 (End of February):**

Cumulative assessment comprised of 20 multiple choice questions and 4 free response questions covering limits, continuity, rules of differentiation, applications of differentiation and the integral. All questions are taken from released AP Exams and are part of the AP Preparation. (Jon Ragowski).
Unit 5: Differential Equations and Mathematical Modeling (4 weeks)

✓ Slope Fields

✓ Antiderivatives
  ▪ Indefinite integrals
  ▪ Power formulas
  ▪ Trigonometric formulas
  ▪ Exponential and Logarithmic formulas

✓ Separable Differential Equations
  ▪ Growth and decay
  ▪ Slope fields (Resources from the AP Calculus website are liberally used.)
  ▪ General differential equations
  ▪ Newton’s law of cooling

✓ Logistic Growth

Unit Expectations:
By the conclusion of unit five, the learner will demonstrate an ability to recognize, construct and solve differential equations using a variety of methods. Students will thoroughly explore math modeling in growth and decay. Specific demonstrations of ability include:

- Define and use appropriate integrals in a variety of applications.
  - Find specific anti-derivatives using initial conditions.
  - Solve separable differential equations and use them in modeling. In particular, study the equation $y' = ky$ and exponential growth
  - Use differential equations within slope fields to solve for general antiderivatives.
  - Apply initial conditions to find specific solution equations.

Authentic Assessment:
Students will gain an understanding of differential equations by using TI-84 software to plot slope fields. Given data concerning the amount of land necessary to provide food for one person and also given the population of the world at different time, students will calculate the rate of growth of the world’s population using growth and decay formulas to calculate when the maximum sustainable population of the earth will be reached. Student will research their methods based on the methods used to calculate herd size via the Department of Agriculture.
Evaluation Criteria:

Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2nd day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.

Benchmark 5 (End of March):

Cumulative assessment comprised of 20 multiple choice questions and 4 free response questions covering limits, continuity, rules of differentiation, applications of differentiation; the integral and differential equations. All questions are taken from released AP Exams and are part of the AP Preparation. (Jon Ragowski).

Unit 6: Applications of Definite Integrals (3 weeks)

✓ Integral as net change
  ▪ Calculating distance traveled (particle motion)
  ▪ Consumption over time
  ▪ Net change from data

✓ Area between curves
  ▪ Area between a curve and an axis
    • Integrating with respect to x
    • Integrating with respect to y
  ▪ Area between intersecting curves
    • Integrating with respect to x
    • Integrating with respect to y

✓ Calculating volume
  ▪ Cross sections
  ▪ Disc method
  ▪ Shell method

Unit Expectations:
By the conclusion of unit five, the learner will demonstrate an ability to recognize, construct and appropriate integrals to solve area, volume and accumulated change applications. Specific demonstrations of ability include:
• Define and use appropriate integrals in a variety of applications.
  o Interpret the integral of a rate of change to give accumulated change.
  o Find the area of a region.
  o Find the volume of a solid with known cross sections.
  o Find the average value of a function.
  o Find the distance traveled by a particle along a line.

Authentic Assessment:

Students will gain an understanding of the application of the integral. They may demonstrate this understanding via a variety of tasks including:

Students are given a table of values showing time elapsed and the download speed of a file from the internet in kb/sec. Using this data of rate of change versus time, the students will compute midpoint Riemann sums to approximate the size of the downloaded file. Then they will calculate acceleration at a specific time, approximate the size of the downloaded file by using a trapezoidal sum, and the average download speed. They will the document their results.

Students will be given a doughnut which has been sliced in half and a sheet of graph paper. Students will then “sketch” the cross-sectional view of the doughnut, calculate an accurate equation, and then compute the volume of the entire doughnut when their cross section is rotated about an axis that will produce the doughnut’s shape.

Students will be given a Styrofoam cup and a taped measure. They must then devise a way to find the volume of the cup by writing a function to represent the cup, then writing an integral for finding the volume of a solid of revolution, then calculating the volume. Students must also convert their answer into ounces to verify their results.

Evaluation Criteria:

Students are evaluated by results of homework and classwork, labs, a minimum of 2 formal quizzes and a formal assessment at the conclusion of the unit. Accommodations: Students are provided 2 class periods to take the formal assessment (first day – calculator, 2nd day – no calculator) but may stay after school either of those days if additional time is required at the discretion of the student. Additional time is without calculator.

Unit 7: Review/Test Preparation (time varies based on AP Exam date)

✓ Multiple-choice practice (Items from past exams along with the scoring rubrics for the last 7 years are used as well as items from review books purchased over the years.)
Test taking strategies are emphasized
Individual and group practice are both used

☑️ Free-response practice (Released items from the AP Central website are used liberally.)
  - Rubrics are reviewed so students see the need for complete answers
  - Students collaborate to formulate team responses
  - Individually written responses are crafted. Attention to full explanations is emphasized

Unit 8: Parametric and Polar Systems (as time permits)

☑️ Define parametric equations
  - Graph curves parametrically using a graphing calculator
  - Solve application problems using parametric equations

☑️ Define polar equations
  - Convert points and equations from polar to rectangular coordinates and vice versa
  - Graph polar equations
  - Determine the maximum r-value and symmetry of a graph

This optional unit is interesting and fun for the student. We will be using Unit 10 from The Precalculus Manual (www.mastermathmentor.com) for guided notes and practice as this area of study is completely new to the student. Assessment is by classwork, homework and calculator lab only. Parametric and Polar systems are not usually introduced until 2nd year Calculus at the college level but, nonetheless, many engineering and physics students are forced to use them much earlier in their college careers.

Final Exam Preparation

In addition to formal review and individual preparations, students will have their choice of the following:

Culminating Project 1 (ongoing through unit 7):
Each student in the class will pick a topic we have covered in the previous units and teach a review lesson about it to the class. Student presentations will include a written lesson plan (to turn in), an oral presentation, a mini-quiz for the end of class, and an answer key to that quiz with detailed explanations, in complete sentences, of each problem solved.

Culminating Project 2- The Sudoku Challenge:
Students work in small groups to develop their own system of notation for describing, solving, and writing theorems about the Sudoku puzzle. Students get the opportunity to see how difficult it can be to come up with a concise, clear, intuitive, and suggestive way of symbolizing a problem.
Students also get to do their own “mathematical work”, which is presented to the teacher as a written project for a grade and to their classmates as oral presentations.

**Culminating Project 3- Iron Chef:**
Using their favorite family recipe, students will rewrite the instructions in calculus. They must have at least one original problem from each of the units. They will provide a copy of the “calculus” recipe and may choose to prepare the dish as part of their class presentation.

All projects will become part of a class notebook.